Continuing on our quest to master coordinate geometry, let's look at a couple of Data Sufficiency questions today. The first question uses a great little concept. The second question has a very important takeaway that we all know but hardly ever implement.

Question 1: In the xy-coordinate plane, line l and line k intersect at the point (5, 4). Is the product of their slopes negative?

1) The product of the x-intercepts of lines l and k is positive.
2) The product of the y-intercepts of lines l and k is negative.

Solution:

Many people start drawing lines to figure out the answer here. Most are able to get the correct answer but they just take far too much time trying out various cases – making l with x intercept positive, k with x intercept positive, then l with x intercept negative etc. What we forget here is something we know intuitively but never use:

Slope of a line = – (y intercept)/(x intercept)
If you are wondering why it is so, think what 'intercept' represents…
The point of x intercept is (X, 0) (where y co-ordinate is 0). We say the x intercept is 'X'.
The point of y intercept is (0, Y) (where x co-ordinate is 0). We say the y intercept is 'Y'.

So the line passes through these two points: (X, 0) and (0, Y)

If we know two points through which the line passes, we know we can find its slope. If the two points are (x1, y1) and (x2, y2), we know the slope = (y2 – y1)/(x2 – x1).

Here the two points that we have are (X, 0) and (0, Y)
So slope = (Y – 0)/(0 – X) = -Y/X

If you remember, Y was the y intercept and X was the x intercept. Therefore, slope is given by –(y intercept)/(x intercept) in terms of intercepts.

Let's use this discovery to quickly solve the question now.

Let us say that the x intercept of line l is xl and y intercept of line l is yl. Also, let us say that x intercept of line k is xk and y intercept of line k is yk
Slope of line l = -yl/xl
Slope of line k = -yk/xk

Product of the slopes of lines l and k = (yl*yk)/(xl*xk)

Statement 1: The product of the x-intercepts of lines l and k is positive.

We are given that xl*xk is positive. But we have no information about yl*yk. Hence, this statement alone is not sufficient.

Statement 2: The product of the y-intercepts of lines l and k is negative.

We are given that yl*yk is negative. But we have no information about xl*xk. Hence, this statement alone is not sufficient.

Using both together, we know that xl*xk is positive and yl*yk is negative. Hence (yl*yk)/(xl*xk) must be negative.
Sufficient.

Answer (C).

Concept to Remember: Slope of a line is given by (–y intercept/x intercept) in terms of intercepts.

Let's go on the next question now.

Question 2: On the xy-coordinate plane, a triangular region is bounded by the lines y = 3, x = -6, and y = cx + d. One vertex of this region is (-6,0). What is the perimeter of this region?

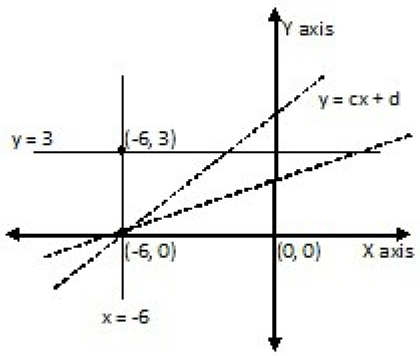
Statement 1: d = 3

Statement 2: c = 1/2

Solution: We all know that a Data Sufficiency question is just that – a sufficiency question. You don't actually need to solve it (with a few exceptions). All you need to do is find out whether the data is sufficient to solve it. If you forget this critical point during the exam, it will be many minutes before you finally mark (D) as your answer for this question and move on. But if you do remember this point then it will take you less than a minute to correctly reach the same answer. If you don't believe me, go ahead and solve it. Clock the time you take. Then think how you could have figured out the answer without actually solving it.

Let me give you my train of thought:

Let's first see what the triangle looks like.



The dotted lines show the third side. Since we do not know the values of c and d, we cannot plot the line accurately. It could look like either of the two dotted lines shown or something similar. The point (-6, 3) will be a vertex of the triangle since it is the point of intersection of x = -6 and y = 3. Since (-6, 0) is given as a vertex too, the line y = cx + d must pass through it.

To get the perimeter of the triangle, you need to know the length of the sides. To know the length of the sides, you need the co-ordinates of the vertices. (If you know the two vertices (x1, y1) and (x2, y2), the length of the side is given by √((x1 – x2)^2 + (y1 – 2)^2).

To know the co-ordinates of the vertices, you need to know the equations of all the lines. (When you know the equation of two lines, you can find their point of intersection by solving them simultaneously.)

You know the equation of two lines: y = 3 and x = -6. But you do not know the equation of the third line, y = cx + d. All that is missing to get the perimeter of the triangle is the equation of this line. c is the slope of this line and d is the y intercept. We see from the diagram that this line passes through (-6, 0) so we already have one point through which it passes. Let's look at the statements now.

Statement 1: $d = 3$

This statement tells us that the y intercept is 3. This means that this line passes through (0, 3). So now we know 2 points through which this line passes (-6, 0) and (0, 3). Hence we can find the equation of the line. This statement alone is sufficient to get the perimeter of the triangle.

The equation of a line when two points, (x_1, y_1) and (x_2, y_2), through which it passes are given, is $y - y_1 = [(y_2-y_1)/(x_2 - x_1)](x - x_1)$

Statement 2: $c = 1/2$

This statement tells us that the slope of the line is 1/2. Now we know a point through which this line passes (-6, 0) and its slope. The slope and a point are sufficient to find the equation of the line. This statement alone is sufficient to get the perimeter of the triangle.

The equation of a line when a point, (x_1, y_1), through which it passes and the slope, m, are given, is $y - y_1 = m (x - x_1)$

Since both statements are individually sufficient to get the perimeter of the triangle, the answer is (D).

Takeaway: Data Sufficiency questions needn't be solved. You just need to know whether the given data is sufficient to solve!